Approximate calculations of the transient electromagnetic response from buried conductors in a conductive half-space

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ABSTRACT

The transient electromagnetic (TEM) response from a conductive plate buried in a conductive half-space and energized by a large-loop transmitter is investigated in a heuristic manner. The vortex and galvanic components are each calculated directly in the time domain using an approximate procedure which ignores the electromagnetic coupling present in the complete solution. In modeling the vortex and galvanic current flows, the plate is replaced with a single-turn wire loop of appropriate parameters and a distribution of current dipoles, respectively. The results of calculations of the transient magnetic field at the surface of the earth are presented for a few selected cases of practical interest.

The relative importance of the vortex and galvanic components varies with the half-space resistivity. The vortex component dominates if the half-space is resistive, in which case free-space algorithms suffice for numerical modeling. Furthermore the measured responses give much useful information about the target, and large depths of exploration should be achieved. As the half-space resistivity decreases, a significant half-space response is observed, caused by currents induced

in the half-space itself. This response can be very large. Spatial variations in it caused by relatively small changes in resistivity, i.e., geologic noise, obscure the response from deep targets making them difficult to detect. The effect of the half-space is also to delay, distort, and reduce the vortex component in comparison with the free-space response.

The behavior of the galvanic component is determined by the half-space current flow. The presence of this component explains the large enhancement of overall target response seen at early times over relatively resistive ground and the departure from an exponential decay seen over more conductive ground, again with respect to responses predicted by free-space modeling. In more conductive ground the galvanic component completely dominates the vortex component, resulting in the loss of useful diagnostic information. Although target location and depth can still be determined, target shape and orientation are poorly defined. Because of galvanic current saturation good conductors are difficult to distinguish from poor ones.

INTRODUCTION

The past few years have seen increasing applications of large-loop transmitter transient electromagnetic (TEM) surveys for mineral exploration in conductive environments. Analysis of survey data indicates that in such environments free-space modeling cannot always be used for interpretation. It is clear that the conductive host medium often exerts a considerable influence on the response of confined targets. For example, at times early in comparison with the inductive time constant τ of a plate-like target the measured response may be an order of magnitude larger than that generated by free-space vortex currents alone, while at late times the characteristic free-space exponential decay may not be observed at all.

The current state-of-the-art in numerical modeling is such that it is impossible to calculate accurately the TEM response directly in the time domain for general three-dimensional (3-D) bodies located in a conductive host and arbitrarily situated with respect to the transmitter. For certain cases, however, analytical data are available; for example the transient response for a sphere is given by Singh (1973) and Lee (1983). Lajoie and West (1976) carried out computations for a plate, often a more useful model for exploration purposes, in the frequency domain; however, their calculations are costly. While translation to the time domain by inverse Fourier transformation is quite feasible (Lamontagne, 1975), routine application would be prohibitively expensive. It is also difficult to employ

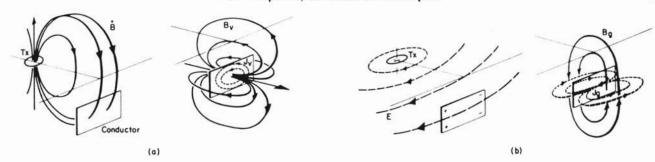


Fig. 1. EM fields and currents about a conductive plate in a conductive half-space. Solid lines indicate magnetic field components, dashed lines indicate electric field components and currents. (a) Illustrates the generation of vortex currents J_v by inductive coupling with the component of $d\mathbf{B}/dt$ normal to the plane of the plate. \mathbf{B}_v is the secondary magnetic field due to the vortex currents. (b) Illustrates the generation of channeled currents J_g by galvanic coupling with the component of the primary electric field \mathbf{E} in the plane of the plate. \mathbf{B}_g is the secondary magnetic field due to the galvanic current.

physical "tank modeling" experiments to determine the response from such targets. The electrical conductivity of available noncorrosive electrolytes is not large enough to prevent sensing the tank boundaries unless the tank is large and the response is measured at very early times.

A complete mathematical treatment of the problem certainly indicates that the target and the host medium interact electrically in a complicated manner. However, for a number of years, and with some degree of success, workers have been stripping away the response of the host medium from the combined target/host response by simple subtraction to obtain the geometric anomaly associated with the target. In an effort to extend at least partially our interpretative capabilities, particularly for targets of large strike-length, this idea of superposition is taken one step further. Not only do we superpose the response of the half-space and the target, but we also represent the target response itself as a superposition of two physically distinct responses-inductive and galvanic. These superpositions will be valid for some target geometries and over certain ranges of time. Studies by March (1953), Singh (1973) and Kaufman (personal communication) of the transient response of a sphere in a whole space support this concept for the sphere. Lajoie and West (1976) explicitly derived the coupled galvanic and inductive equations for a plate in the frequency domain and showed that some of the coupling terms are small.

We are interested in the interpretation of field data collected over base metal mineral deposits. These are typically limited in thickness to a few meters, may have depth and strike extent of the order of several tens or hundreds of meters, and are often steeply dipping. A plate-like model is thus an appropriate representation of an economically relevant target.

Our objective here is to obtain a useful estimate for the galvanic and inductive responses of a conductive plate in a conductive host medium. Transient responses typically have a dynamic range of many orders of magnitude. Consequently, we would consider any prediction from an approximate model that was correct to within a factor of 2 or 3 to be useful, not only for illustrating the basic physics of the problem but also for preliminary interpretation of field data.

The contents of this paper are set out in the following way. First we examine the three factors which contribute to the transient response. These are the direct response from currents

in the half-space itself, the response from vortex currents in the plate, and the response from galvanic currents flowing in both the half-space and the plate. The simple methods for calculating each of these are described and the various approximations are summarized. The final section illustrates the characteristics of these responses as a function of time and modeling parameters.

FACTORS CONTRIBUTING TO THE TRANSIENT RESPONSE

Half-space currents

The transient response of a uniform half-space or layered earth has been extensively covered in the literature (Hoversten and Morrison, 1982; Kaufman, 1978b; Lamontagne, 1975; Oristaglio, 1982; Wait, 1982) and readers are referred to these works and references cited therein for the basic theory. A good physical description of the way in which the currents appear to diffuse into the earth is given by Nabighian (1979). We will include a small number of calculations of the half-space response for comparison with our plate responses.

Vortex currents in the plate

We initially consider a target in free space, as shown in Figure 1a. In the transient method the primary magnetic field **B** is rapidly reduced from a constant value to zero. The emf induced in the target in free space, being proportional to $d\mathbf{B}/dt$ in accordance with Faraday's law, is therefore impulsive; it causes vortex currents \mathbf{J}_{v} to circulate which decay with time. For a finite body, at late time, the decay eventually becomes exponential with a time constant τ_{b} which is characteristic of the body's geometry and conductance. At early times the decay may be represented by a sum of exponential decays (Kaufman, 1978a).

Accurate calculation of the vortex currents and associated secondary fields for plate-like targets in free space can be carried out by a method devised by Annan (1974), which has been incorporated into a computer program developed at the University of Toronto and described by Dyck et al. (1980). The method incorporates the response from both low- and high-

920 McNeill et al.

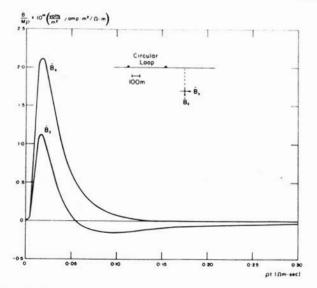


FIG. 2. Time derivative of subsurface magnetic field as a function of (resistivity \times time). **B** is normalized with respect to M, the transmitter dipole moment, and ρ , the half-space resistivity. The normalization allows the curve to be used for any value of resistivity.

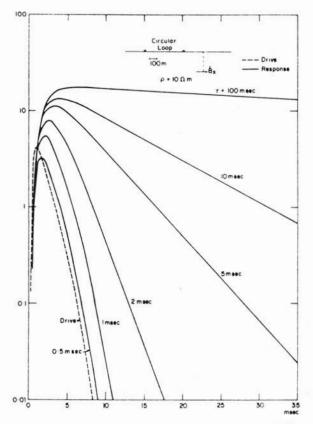


Fig. 3. The relative effect of the broadened and delayed drive on the time behavior of vortex currents. B_x from Figure 2 has been convolved with simple exponential decays, $\exp(-t/\tau)$, for various values of τ . For an infinitely fast impulse all curves would become straight lines intersecting the ordinate at a value of 18 units (the area under the selected drive curve).

order eigencurrents. The latter have short time constants and thus give a significant contribution at early times. At times later than τ_b McNeill (1982) showed that, if the vortex currents are approximated by a single wire-like current loop, a good match to the late-stage response as calculated with the University of Toronto program is still obtained. For times much earlier than τ_b the more accurate program shows that the true response is generally about three times larger than that calculated using the simple wire loop model.

Our target is located in a conductive host medium, which alters the impulsive nature of the induced emf as follows. Immediately after transmitter turn-off, assumed to be rapid, currents flow only on the surface of the half-space and are distributed so as to maintain the magnetic field at each point in the interior at the value which existed before turn-off. With the passage of time, currents are generated farther inside the halfspace as described by Nabighian (1979), and it is the time derivative of the relatively slowly varying magnetic field from these currents which now induces vortex currents in the target. To illustrate this effect, a plot of the variation of dB/dt at a point located 500 m from the center of a circular loop transmitter and at a depth of 300 m within a conductive half-space is shown in Figure 2. It is evident that both the horizontal and vertical components are delayed and broadened in comparison with the infinitely narrow impulse which would occur at time t = 0 in free space. Furthermore, as indicated by the normalization used in the figure, the area under each curve is not a function of the half-space resistivity; a more conductive environment simply lengthens the duration of the pulse and proportionately reduces the amplitude. Calculations for other locations in the half-space show that, as expected, both the delay and the broadening increase with radial distance and depth, so that for large targets these features will vary with position

Apart from the delay and broadening another modification of $\dot{\mathbf{B}}$ is caused by the conductive half-space. At the observation point of Figure 2, the primary vertical magnetic field that would be calculated in free space is essentially zero (Macnae, 1980), whereas the amplitude of \dot{B}_z in the figure is comparable to the amplitude of \dot{B}_z . However, the areas under the positive and negative excursions of \dot{B}_z are equal, so the net vertical component of the drive is indeed zero for targets with large time constant τ_b . However, for targets with short time constant the presence of this drive component substantially alters the way in which measured responses vary with target dip angle as compared with free space.

In general the drive delay and broadening results in a response which is delayed and reduced in amplitude by an amount which depends upon the target time constant. To illustrate this effect for a vertical target, Figure 3 displays the results of convolving \vec{B}_x from Figure 2 with the simple exponential decay exhibited by the current in a single wire loop, for various values of time constant τ , i.e., of convolving \vec{B}_x with $\exp(-t/\tau)$. Since the area under the selected drive curve was 18 units, an infinitely fast impulse would have resulted in a response given by 18 $\exp(-t/\tau)$. For the extended pulse at early times the current follows the drive for all τ . At later times for values of τ large in comparison with the drive duration the characteristic exponential decay is eventually seen, whereas for small values of τ the current continues to follow the drive for all time. Furthermore, for τ large compared with the drive dura-

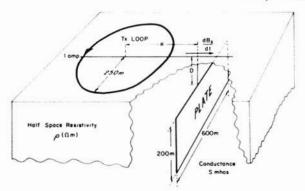


Fig. 4. Geometry for calculations of half-space, vortex, and galvanic current contributions to B_x shown in Figures 5-7. Calculations were carried out for R = 400, 550 m; D = 100, 200, 300 m; conductance = 30, 100, 300 S; $\rho = 10$, 100, 1000 Ω -m.

tion the peak amplitude is still approximately 18 units, but for smaller τ the peak amplitude decreases with decreasing τ. The transient vortex current response in a plate in free space can be represented as the response from an infinite sum of eigencurrents with progressively shorter time constants. If the same plate is embedded in a conductive host, only the eigencurrents with time constant greater than the drive duration will have an amplitude approximately equal to their free-space value; the amplitude of eigencurrents of shorter time constant will be selectively reduced. Thus for conductive ground, at measurement times comparable to the drive duration, it will be impossible to match the inductive transient response of a plate with any free-space plate model; for times later than the drive duration, the simple wire loop approximation which incorporates only the largest time constant of the complete model will be adequate, provided that the target is not situated too near the transmitter. This argument is the basis for our adoption of the simple loop model for computing the approximate vortex responses later in this paper. In practical terms, for the plate models that we will be considering, it will be seen that the error from the simple loop model is significant only when the vortex current is found to be of negligible importance in the overall response.

Galvanic currents

The transient electric field in the half-space, shown in Figure 1b, has a behavior which depends upon the conductivity of the half-space and the distance to the source. The current which flows is related to this field by Ohm's law. The current is diverted in the vicinity of the plate by the action of a secondary electric field, which in turn is caused by charges impressed on the surface of the plate. This process is variously referred to as current channeling, current gathering, or current streaming. The galvanic current flow J_g is sketched in Figure 1b. It can be thought of as an exchange of current between the plate and half-space (Edwards, 1974, 1984; Kaufman and Keller, 1981). Its strength depends upon the strength of the exciting electric field, the conductivity contrast between the half-space and the plate, and the dimensions of the plate.

Stefanescu (1958) introduced the concept of a current dipole to represent current exchange between a conductive body and its environment. An infinitesimal current dipole consists of a point current source and a point current sink linked by a very short current element. Many such dipoles may be assembled to represent the current channeling by a plate (Edwards, 1984; Nabighian et al., 1984). The current elements lie in the plane of the plate; their values are determined by means of an integral equation formed by equating the tangential electric field just inside and outside the plate. The equation includes the fields of the distributed dipoles and also the component of the exciting electric field impressed in the plane of the plate.

Our approach to modeling the galvanic response for a plate target has been to calculate for each selected instant of time (after transmitter turn-off) the undisturbed electric field in the vicinity of the target, and to use this as the exciting field for a static modeling procedure. The galvanic response at any time is thus assumed to be caused by the regional electric field at that time.

SUMMARY OF APPROXIMATIONS

In carrying out the calculations, many approximations, both numerical and physical, have been made. Physically, the galvanic and vortex currents are treated independently, whereas in fact they interact. The Green's functions used in calculating the vortex and galvanic responses are free-space and static halfspace functions, respectively; attenuation and delays due to the host rock conductivity, important at early times and large distances, are ignored. The vortex currents are represented by a simplified free-space model which is accurate only at late time, but in accordance with our definition is useful at earlier times.

RESULTS

A restricted number of cases were chosen on the basis of relevance to exploration. The geometry is shown in Figure 4. The transmitter loop has a radius of 250 m. The vertical conductive plate has a strike length of 600 m and a depth extent of 200 m; it therefore represents a large conductor. The depth to top was varied from 100 to 300 m; the radial distance from the transmitter center was either 400 m or 550 m, and the half-space resistivities were 1000, 100, and 10 Ω -m. Plate conductances were selected as 30, 100, and 300 S, representing relatively conductive targets. While all results will be discussed, plots are shown only for the plate at a depth of 200 m and at a radial distance of 400 m.

Since a coil is employed to measure the transient response, we plot the time derivative of the magnetic field. Figures 5, 6, and 7 show decay curves of the time derivative of the horizontal component \vec{B}_x , normalized with respect to the transmitter current. The decays are calculated for a point on the surface located directly above the vertical plate, where both the galvanic and vortex contributions for this component peak. Our choice of the horizontal component is not just for mathematical convenience. It can be argued that its measurement in the field has several advantages which complement measurement of the vertical component. The minimum sferic noise level which might be expected for the horizontal component \vec{B}_x is indicated on the figures; it has been adjusted on the basis of a transmitter

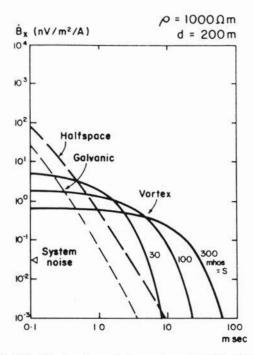


Fig. 5. Calculated values of the various contributions to B_x (normalized with respect to transmitter current) as a function of time for half-space resistivity of 1000 Ω -m. Depth to top of plate = 200 m. Transmitter current turn-off is a linear ramp of duration 300 μ s. System noise level indicated is the minimum expected for horizontal component. Target time constants are 1, 3, and 10 ms. Note that the galvanic response is the same for all three plates.

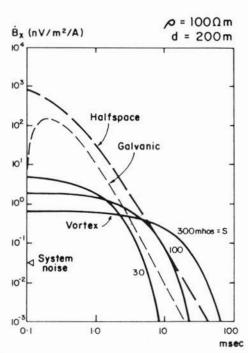


Fig. 6. Calculated values of the various contributions to B_x (normalized with respect to transmitter current) as a function of time for half-space resistivity of 100 Ω -m. All other parameters as for Figure 5. Actual vortex responses will show a crossover at time $\cong 0.4$ ms.

current of 20A. Figures 5, 6, and 7 are arranged in order of decreasing half-space resistivity. In each figure the contributions from the half-space, the galvanic currents, and the vortex currents are shown separately. Note that all calculations were carried out for a linear-ramp transmitter current turn-off time of 300 µs, which modifies the response at earlier times.

The classical response for the half-space can be seen in full only for the relatively conductive case of Figure 7. An initial negative response at early time is followed by a positive peak and finally a t^{-3} power-law decay. For the less conductive half-spaces of Figures 5 and 6 the initial negative portion is shifted to times earlier than plotted. The maximum amplitude of the half-space response is strongly dependent upon the half-space conductivity, yet for all cases considered this response still dominates for at least a significant portion of the normal measurement time range.

The galvanic component clearly behaves with time in a similar fashion to the half-space component. An initial negative portion is once again followed by a positive peak which subsequently decays as $t^{-7/2}$, i.e., at a slightly faster rate than the half-space response. The crossover in the galvanic component is delayed relative to that for the half-space: the zero crossing in B_x for the galvanic component corresponds to the time of the maximum electric field at the plate, which occurs at a later time than the maximum in the horizontal magnetic field at the surface. In the two more resistive half-spaces (Figures 5 and 6), the galvanic component from the plate is independent of plate conductance; in the most conductive ground only slightly dif-

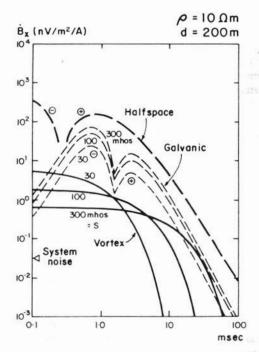


Fig. 7. Calculated values of the various contributions to B_x (normalized with respect to transmitter current) as a function of time for half-space resistivity of 10Ω -m. All other parameters as for Figure 5. Actual vortex responses will show a crossover at time $\cong 4$ ms. Note that the galvanic component now depends on plate conductance. Both the galvanic and half-space contributions exhibit a sign reversal.

ferent curves are seen. In general the galvanic component increases with increasing channeling number α as $\alpha/(1+\alpha)$, where $\alpha = \rho S/\ell$, ℓ is the conductor strike length and ρ the half-space resistivity (Nabighian et al., 1984).

The galvanic component is thus usually saturated with respect to the plate conductance, so that unlike the vortex response, the galvanic response is insensitive to target conductance, poorly conductive targets yielding a response that is nearly as large as highly conductive targets. Furthermore the nature of the transient decay itself is controlled by the halfspace; it too is independent of plate conductance. The late-time shape of the vortex responses, being a simple exponential decay, is quite different from the quasi power-law decays of the other two components. Because measurement is made of B, the plotted plate responses take the form $\tau^{-1} \exp(-t/\tau)$, which accounts for the reduction in initial amplitude with increasing plate conductance (t being proportional to S). At early times it must be kept in mind that the plotted vortex responses are inaccurate principally due to the delayed and broadened drive referred to earlier. We measure B which is proportional to the time derivative of the plate vortex currents illustrated in Figure 3. The derivative exhibits complex behavior at early time, changing sign before becoming exponential. The crossover will occur at approximately 4 ms, 400 μ s, and 40 μ s for $\rho = 10 \Omega$ -m, 100 Ω -m, and 1000 Ω -m, respectively. At these times for each case the plotted vortex component in Figures 5-7 is negligible compared with the half-space and galvanic components so that the error is of no consequence.

It is clear from Figure 5 that, at least in resistive ground, free-space modeling can be quite appropriate, since at intermediate to late times the vortex component dominates. However at early time the galvanic component is the larger, apparently corroborating our field experience in resistive terrain. The detectability of the vortex component decreases systematically with host resistivity, until in 10 Ω -m material at no time is the vortex component significant. Whether the vortex response will ever dominate is dictated almost entirely by the target time constant; since the galvanic component decreases rapidly with time, a moderate increase in time constant greatly increases the ratio of vortex to galvanic response. At very late times the vortex response decays much more rapidly than the galvanic response so that the latter again dominates. In fact, although it is not indicated in the figures, it can be shown that for very late times the vortex response shifts from an exponential to powerlaw decay as the tail of the drive emf generated by the decaying currents in the host becomes more important than the original impulse. Such a change in response will probably occur outside of the measurement range.

Calculations for plates at depths of 100 and 300 m add little, except to show that the galvanic component decreases slightly more slowly with depth than the vortex component. For plates located at a radial distance of 550 m the vortex component decreases by a factor of approximately 2, whereas the galvanic and half-space contributions increase slightly.

Unfortunately, loss of the vortex component is a serious matter, since this quantity yields useful diagnostic information about the target conductance, depth to top, depth extent, and dip angle. Nevertheless, simple target detection with an estimate of depth is important. The galvanic component can certainly supply the location of the target and depth to the "centroid," both of which are obtained from inspection of the spatial

variation of the field components along the survey profile, an example of which is shown in Figure 8. Surprisingly, profiles of the vortex and galvanic responses for vertical plates bear considerable resemblance to each other since, as shown by Edwards (1974), the galvanic response from a finite body also has negative wings, now caused by the return current flow in the host medium. The galvanic profile varies somewhat with time due to variation of the spatial distribution of the primary electric field at the plate.

Without a good estimate of the accuracy of our calculations it is difficult to determine with certainty the maximum depth to which the modeled plates could be detected in a conductive half-space on the basis of the galvanic component alone. However, examination of Figure 7 suggests that in 10 Ω -m ground the more conductive plates should be detectable to depths of the order of 200 m if we assume that the galvanic response is detectable against a half-space response which is four times larger. This is much shallower than could be reached in resistive ground using the vortex component. Larger transmitters will not extend the exploration depth in a conductive environment since the limitation is geologic noise arising from relatively minor resistivity variations in the host itself.

CONCLUSIONS

The transient electromagnetic (TEM) response from a conductive plate buried in a conductive half-space and energized

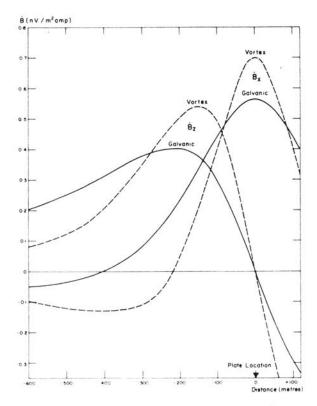


FIG. 8. Profiles of vortex and galvanic components of $\dot{\mathbf{B}}$ over the plate of Figure 4 at a depth of 200 m. Plate conductance = 100 S; half-space resistivity = 100 Ω -m; time = 3 ms. Location of vertical plate is indicated by arrow.

924 McNeill et al.

by a large-loop transmitter has been investigated in a heuristic manner.

The half-space, vortex, and galvanic components were each calculated directly in the time domain using simple rapid computer algorithms. The numerical procedures adopted are themselves approximations, and in addition the EM coupling between the vortex and galvanic current flows is ignored. Consequently, the methodology is not generally valid but should be useful if applied with discretion.

The computed transient magnetic field at the surface of the earth was presented for a small selection of cases of practical interest. The relative importance of the vortex and galvanic components varies with the half-space resistivity. The vortex component dominates for at least a portion of the transient response if the half-space is resistive. In this case free-space computer algorithms can be used for numerical modeling and the large depths of exploration that they predict should be realized. Furthermore the vortex response is strongly dependent upon target size and orientation, and also on target conductance, so that useful diagnostic information is generated.

As the half-space conductivity increases, the half-space response becomes very large and spatial variations in it due to small changes in resistivity can reduce the depth of exploration. Further effects from the half-space are the delay and broadening of the vortex current drive emf and alteration of the spatial distribution of the drive compared with free space.

The time behavior of the galvanic component is determined by the current induced at depth in the half-space and is very different from the vortex component. It is relatively unaffected by target conductance. The presence of the galvanic component explains the large enhancement of overall target response seen at early times over relatively resistive ground, and the departure from an exponential decay seen over more conductive ground, again with respect to responses predicted by free-space modeling. In ground of intermediate conductivity it will be difficult to distinguish between galvanic and vortex responses. In more conductive ground the galvanic component can totally dominate the vortex component. The result will be a substantial loss of information. For a target whose depth extent is comparable with its depth of burial, target dimensions and orientation will be poorly defined, although the location and approximate depth can still be determined provided that the interpreter realizes that galvanic currents are causing the response. More importantly it will be difficult to distinguish the better conductors from the poor ones since galvanic saturation causes both to yield essentially equal response.

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