GEONICS LIMITED
1745 Meyerside Dr. Unit 8 Mississauga, Ontario Canada L5T 1C6

Tel: (905) 670-9580
Fax: (905) 670-9204
E-mail:geonics@geonics.com
URL:http://www.geonics.com

Technical Note TN-37

TIME-DOMAIN RESPONSE OF A MAGNETICALLY SUSCEPTIBLE SOIL

J. Duncan McNeill

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Time-domain Response of a Magnetically Susceptible Soil

In a previous Geonics Limited Technical Note (TN 36: The Magnetic Susceptibility of Soils is Definitely Complex) we described a 'finite discrete time-constant' model to simulate the magnetic susceptibility of soils to sinusoidal excitation over a wide band of frequencies. This model can also be used to describe the step-function response in the time-domain.

In TN36 it was stated that Néel (1949) showed that τ, the observed time-lag, or 'time-constant' in the response from a single-domain mineral grain, is a strongly varying function of the domain volume V, given by

$$\tau = A \exp(V J_s H_c / 2kT) \qquad (1)$$

where A = constant

V = grain (domain) volume

J_s = spontaneous magnetic moment of the grain

H_c =coercive magnetic force of the grain

k =Boltzmann constant

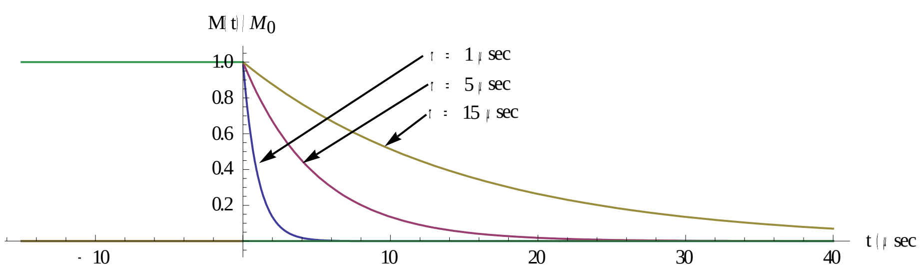
T =absolute temperature

and $(V J_s H_c / 2)$ is the energy required to overcome the energy barrier.

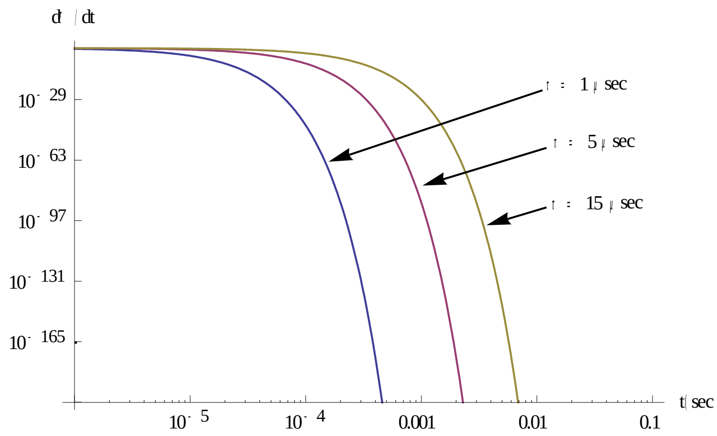
If an assemblage of SD grains all having the same time-constant τ has been magnetized by a constant uniform primary magnetic field of strength M_0 which is abruptly terminated at time t =0, Néel states that the remanent magnetization of the assemblage effectively decays with time t as

$$M(t) / M_0 = \exp(-t / \tau\ ). \qquad (2)$$

The following figure shows this transient behavior (i.e. the delayed response) for three SD assemblages with different time-constants of 1, 5 and 15 μsec.



The next figure shows the same data replotted on a log-log plot for a comparison with further calculations.



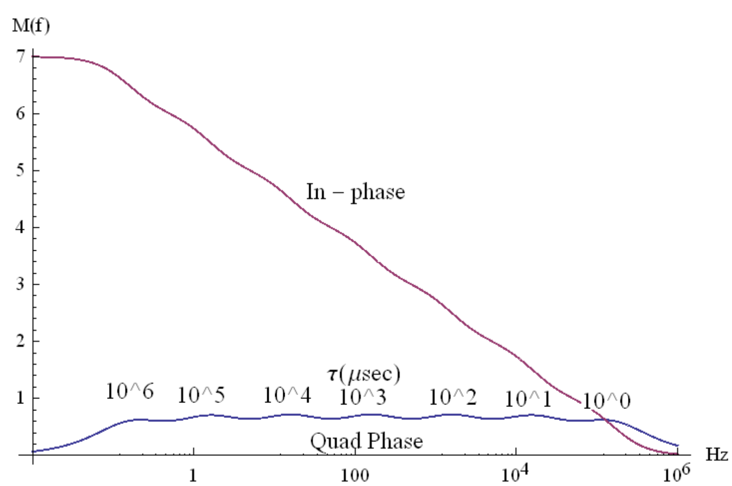
Mullins and Tite (1973) explored the time response of the remanent magnetism σ_r for the Néel theory of an infinite assemblage of grains when a primary magnetic field of strength h_0 is abruptly terminated at $t=0$.

They showed that (using Néel's notation)

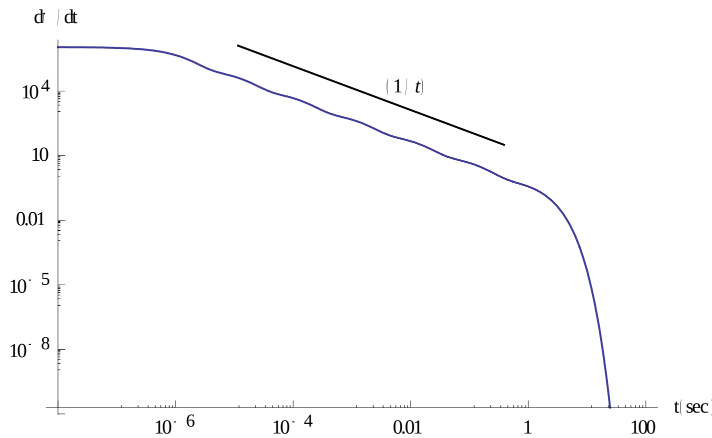
$$\chi_q = \frac{-\pi}{2h_0}\frac{\partial \sigma_r}{\partial \log t} = \frac{-\pi t}{2h_0}\frac{\partial \sigma_r}{\partial t}, \qquad \therefore \quad \frac{\partial \sigma_r}{\partial t} = -\frac{2h_0 \chi_q}{\pi t} \tag{11}$$

and since χ_q, the quadrature component of the frequency-domain susceptibility is a constant (as shown in the figure below), $\partial \sigma_r / \partial t$ must decay as $(1/t)$.

The next figure shows the frequency response of our 'finite discrete time-constant' susceptibility model. It consists of the sum of the responses from an assemblage of seven different grain sizes, the smallest with time-constant 1 μsec, each larger size with time-constant greater by a factor of ten, and the largest with time-constant $\tau = 1$ sec.

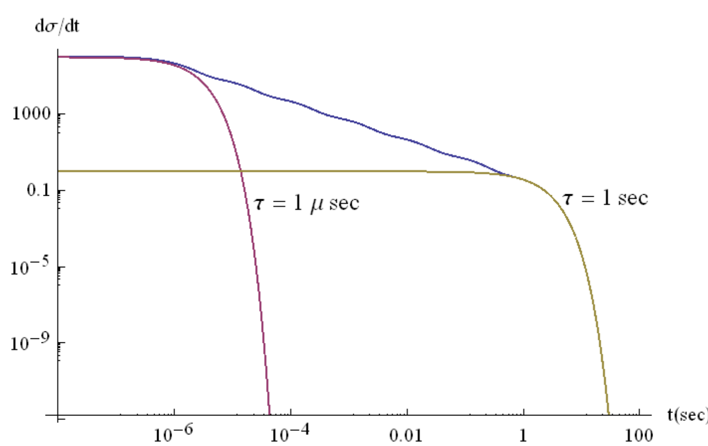


The plot below shows a log-log plot of the calculated time-domain response for our model of the seven time-constant assemblage described above. As for the frequency-domain plot, but now for the time-domain, this plot was generated simply by summing the total time-domain response, now from a series of $(1/\tau)\exp(-t/\tau)$ time responses, one for each of the same seven time–constants used above (the $(1/\tau)$ comes from the fact that an electromagnetic system measures the time-derivative of each of the responses from the different time-constants).



We see that the overall response is divided into three stages. The very early stage, in which the response varies very slowly with time, is simply the initial stage, plotted on a log-log plot, of the exponential response from the shortest time-constant of 1 μsec (along with much smaller contributions from the remaining larger time-constants since their initial amplitudes are multiplied by $(1/\tau)$). The intermediate time stage is the total response from all of the time-constants, each exerting some influence. In accord with the calculations from Mullins and Tite the response at this intermediate time is seen to be described by $(1/t)$. Finally, at latest time the response is totally dominated by the longest time constant, with the result that the time decay is again exponential.

The various features are illustrated in the next figure, which shows the contribution of the earliest and latest time-constants to the overall response shown in the figure above.



Several papers have reported measurements of the rate of time decay of soil samples (Dabas et al. (1992), Dabas et al. (1993)). Even after careful correction for transmitter turn-on effects (which can be significant in view of the slow nature of a (t^{-1}) response) to the best of our knowledge none have shown a true (t^{-1}) response.

Summary and Conclusion

Using the same approximation to duplicate the time-response calculations for an assemblage of time-constants we have shown that the Néel theory prediction for (t^{-1}) time response becomes invalid if there is a limited range of time-constants (or too limited a time-range for a continuous distribution) under which conditions the response becomes exponential at early and late times.

Note that the time/frequency range used above was one-million to one in order to separate out the various stages of the time decay. To the best of the author's knowledge there are no measurements covering such an extended time or frequency range and thus no guarantee that the amplitudes of the different time-constant components are all equal as was assumed above.

Detailed examination at the late-time transition point between (t^{-1}) response and exponential response shows that there is a significant time range, of about a decade in time, where the response is neither (t^{-1}) nor truly exponential. It may be that it is partly this factor that has prevented measurement of a truly (t^{-1}) response.

Bibliography

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